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DRIFT AND OSCILLATORY MOTION OF A VERTICAL CYLINDER ON INTERNAL WAVES

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The behavior of a floating body on surface waves receives much attention in ship hydrodynamics, while relatively few scientific investigations are devoted to the problem on the motion of an immersed body of neutral buoyancy under the action of internal waves in a fluid that is stably stratified in density. This problem has a number of specific features attributed to the fact that the restoring buoyancy force is relatively small under conditions that are important in practice.

The present paper is a continuation of [1] and is aimed at supplementing the available experimental information on the drift and oscillatory motion of a body on internal waves. Here, as in [1], we consider the drift and oscillatory motion of one of the simplest two-parameter bodies, namely, a truncated cylinder. The basic difference is that at rest the longitudinal axis of the cylinder is oriented vertically rather than horizontally, as in [1]. The theoretically and experimentally obtained information on the behavior of a vertical elongated body on internal waves is presented in [2–6].

Formulation of the Problem. A tank with a horizontal bottom is filled with a stably stratified fluid to depth H. There is a layer of constant density ρ_1 near the tank bottom, a layer of constant density $\rho_2 < \rho_1$ near the free surface, and an intermediate layer with characteristic thickness 4δ in which the density ρ changes gradually from ρ_1 to ρ_2. By the interface, we mean a surface at which $\rho = \rho_0 = (\rho_1 + \rho_2)/2$. At rest the interface is at $z = z_0$, where z is a vertical coordinate reckoned upward from the tank bottom. The term wave is used below for interface deviation from the equilibrium position.

Variation in the density was small in the experiments: the parameter $\varepsilon = (\rho_1 - \rho_2)/\rho_2 = 0.5\%$. Two more parameters are essential for the effects considered below: acceleration of gravity g and kinematic fluid viscosity ν. The value of ν changed with depth by not more than 3% in these tests. Such a weak variation did not lead to significant effects, and we can consider the value of ν constant in interpreting the test results.

The main cylinder parameters are as follows: diameter D, length l, and intrinsic principal longitudinal moment of inertia I. The submergence h of the center of mass of the cylinder below the interface and the metacentric height e, i.e., the distance between the center of mass and the point of pressure application at rest, also play an important role.

In the experiments, a wavemaker generated plane progressive travelling waves in the tank. The waves became stationary with time in the sense that they propagated at a constant velocity without changing their shape. Within the accuracy of the results presented below, we can assume that the waves were sinusoidal in a steady regime. The angular frequency Ω and the amplitude a are the main parameters of these waves. The wavelength was formed by the dynamic system itself and was related to Ω in a first approximation by the dispersion relation of the linear theory.

A wave breaker in the form of an oblique plate was placed at the end of the tank opposite to a wavemaker. The reflection from the wave breaker did not exceed 0.05 in the wavelength range considered. In addition, the waves were short and decayed considerably because of viscosity. Therefore, the reflected waves were weaker than the fundamental ones by about two orders of magnitude in the vicinity of the cylinder. The fluid viscosity damped rapidly the second and higher modes of natural oscillations excited by the wavemaker.

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As a result, the wave parameters in the vicinity of the cylinder corresponded to the first internal mode of natural fluid oscillations with an error of not larger than 5%.

Secondary flows, which grew with time, occurred in the tank due to nonlinear effects, agitation by the wavemaker, and fluid friction against the walls. The process of development of secondary flows is slow and has a complex character in a stratified fluid. These flows are first located mainly near the side walls and only gradually reach the central region of the tank. In the experiments, the secondary-flow velocities were about 0.005 cm/sec in the vicinity of the cylinder.

Density stratification was created by adding glycerin to water. The molecular diffusivity of glycerin in water is small (of the order of $4 \cdot 10^{-6}$ cm^2/sec). As a result, the density profile was virtually unchanged during one series of tests.

Additional details of the test procedure are given in [1]. The parameters $H = (42.3 \pm 0.1)$ cm, $z_0 = (26.2 \pm 1.2)$ cm, $\varepsilon = 0.005 \pm 0.00005$, $\delta = (2.5 \pm 0.1)$ cm, $D = 4$ cm, $e = (0.01 \pm 0.001)$ cm, $I = (2700 \pm 100)$ g $\cdot$ cm^2, and $\nu = (0.0100 \pm 0.0001)$ cm^2/sec were fixed. Two series of tests were performed with $h = (6.3 \pm 0.05)$ cm (series 1) and (1.1 ± 0.05) cm (series 2). In the first series $l = (19.35 \pm 0.05)$ cm, and in the second $l = (18.95 \pm 0.05)$ cm. Variation in h was achieved by changing l. At the same time, the parameters I and e were varied within the above-mentioned limits.

Auxiliary tests were performed with free oscillations of the cylinder in the absence of waves. The natural frequencies were found to be virtually the same in both series of tests and equal to (0.45 ± 0.01) and (0.35 ± 0.01) rad/sec for vertical and angular oscillations, respectively. In the basic tests, the wave frequency and amplitude were varied in the following ranges: $0.247 \leqslant \Omega \leqslant 0.616$ rad/sec and $0.47 \leqslant a \leqslant 1.75$ cm in the first series and $0.140 \leqslant \Omega \leqslant 0.559$ rad/sec and $0.78 \leqslant a \leqslant 1.88$ cm in the second.

Altogether 25 tests were performed. The results of each test can be regarded as a partial solution of the problem. The motion of a cylinder is the major object of investigation. The motion was filmed at a frequency of one frame per second. To decrease the error attributed to optical distortions, we used a coordinate grid which was placed in the liquid at a distance of approximately 2 cm from the cylinder trajectory. Such a procedure proved to be possible, because this trajectory was two-dimensional. The root-mean-square measurement error was within 2–5% for various motion characteristics. Apart from the motions, we observed visually the flow pattern in the vicinity of the cylinder and the reciprocal influence of the cylinder on the waves. For this purpose, a liquid layer approximately 1.5 cm thick in the vicinity of the line $\rho = \rho_0$ was dyed with ink.

Under experimental conditions, the motion of the cylinder on plane waves was also plane, and only three of six degrees of freedom were realized: longitudinal motion, vertical oscillatory motion, and angular oscillatory motion in the plane perpendicular to the wavefront. The longitudinal motion is of a constant component (drift). The vertical oscillatory motion had a character of oscillations about the initial equilibrium position. In the case of angular oscillatory motion, the cylinder deflected on the average from the vertical.

It was noted in [1] that the drift of a horizontal cylinder toward the wave can occur in the process of approaching a steady motion regime. This effect was not observed for a vertical cylinder, and the average longitudinal motion occurred only in the direction of wave propagation.

The average axial tilt of a drifting cylinder to the vertical depends on many factors and is difficult to predict. On the average, in the studied range of parameters the upper cylinder end was ahead of its lower end.

The velocity field in the wave changes considerably in the vicinity of the cylinder. Figure 1 shows photographs of two typical positions of the cylinder on the internal wave which propagates from right to left. In the incident wave, the boundaries of the dyed layer oscillate in phase, which is characteristic of the first mode of fluid oscillations. This coherence is violated in the vicinity of the cylinder, and varicose distortions of the layer, which are characteristic of the second mode, develop. This means that the cylinder (as any other obstacle) excites higher oscillation modes of a stratified fluid.

Dynamic and diffusive boundary layers, which are marked by the digit 1, are formed on the cylinder. The significant phase shift between the fluid and cylinder oscillations draws our attention. As the cylinder drifts, vortex structures appear in the wake behind it.

Figure 2 shows plots of the wave η, angular oscillatory motion ψ, longitudinal motions ξ, and vertical oscillations ζ of the geometrical center of the cylinder in one of the tests of the first series. We used a fixed

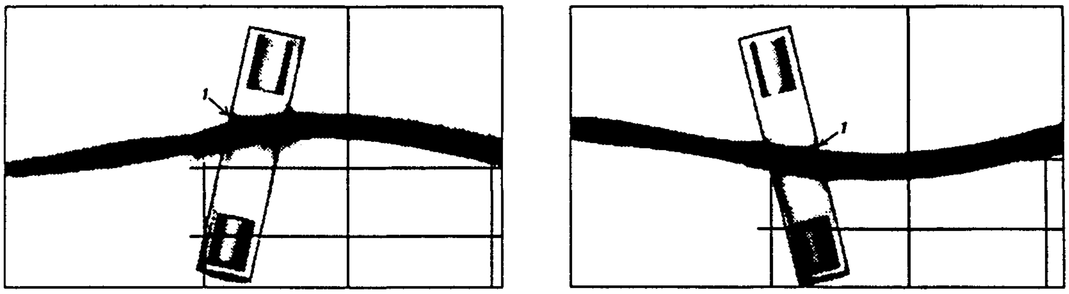

Fig. 1

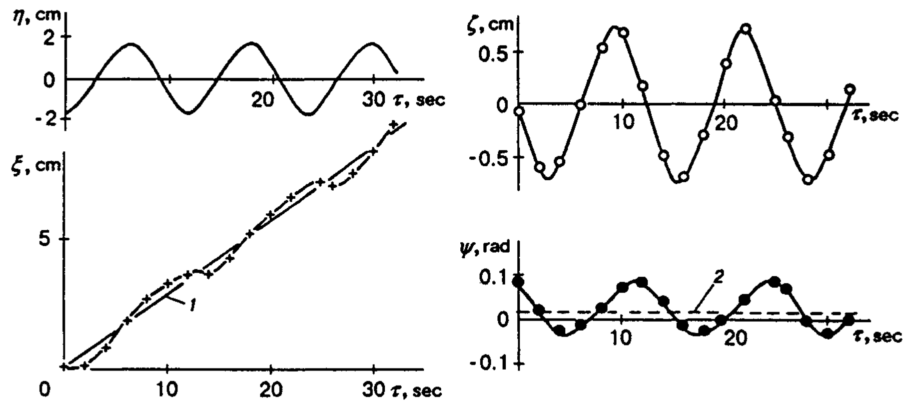

Fig. 2

coordinate system, whose origin is at a certain point of the tank bottom. The x axis is oriented in the direction of wave propagation. The moment of switching on of the wavemaker is taken as the reference time t. The quantity τ in Fig. 2 is related to the time t by the relation $\tau = t - t_0$, where $t_0 = \text{const}$. The value of t_0 was sufficiently large (more than five wave periods) for the wave and cylinder motions to become steady. The quantities ξ and ζ are related to x and z by the relations $\xi = x_* - x_*^0$ and $\zeta = z_* - z_*^0$ (x_*, z_* and x_*^0, z_*^0 are the coordinates of the cylinder center at time t and $t = t_0$). The wave $\eta(\tau)$ was recorded at the tank cross section $x = x_*^0$.

Because of the longitudinal drift of the cylinder, its oscillation frequency Ω^* is smaller than the wave frequency Ω in the fixed coordinate system. In the example considered, $\Omega = 0.532$ rad/sec and $\Omega^* = 0.498$ rad/sec. The slope of the straight curve 1 in Fig. 2 is equal to the mean drift velocity V. In this example, $V = 0.31$ cm/sec, and this is the greatest value for all the partial solutions obtained. The shift of curve 2 in Fig. 2 with respect to the axis τ is equal to the mean angle of tilt of the longitudinal cylinder axis to the vertical; in this example it equals 0.025 rad. The cylinder oscillations decay considerably in phase from the wave. In particular, for vertical and angular oscillations, the phase retardation is close to $\pi/2$.

In oceanology, a vertical truncated cylinder is used as a container of signal and measuring equipment (for instance, the Froude buoy). In this case, it is desirable that the motions of the cylinder be as small as possible. The data obtained allow one to analyze the effect of individual parameters on the motion.

Figure 3 illustrates the effect of Ω on the average drift. It shows the dependences $\zeta(\xi)$ in the fixed coordinate system for $\Omega = 0.532$ rad/sec (wavelength $\lambda = 41$ cm), $a = 0.75$ cm (Fig. 3a) and for $\Omega = 0.331$ rad/sec, $\lambda = 140$ cm, and $a = 0.41$ cm (Fig. 3b) in the first series of tests. The observation time is 32 sec in the first case and 42 sec in the second case. The motion regime is steady. In this example, a decrease in Ω by a factor of 1.6 and in a by a factor of 1.8 led to a more than tenfold decrease in the drift.

It should be noted that in the tests under consideration, the waves were generated in such a way

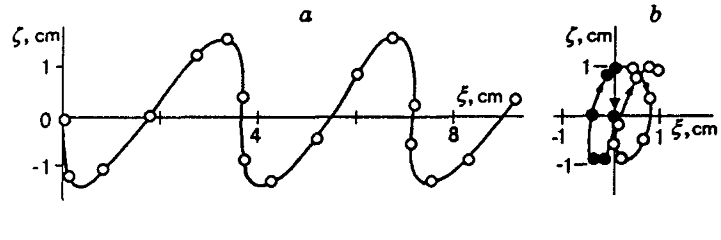

Fig. 3

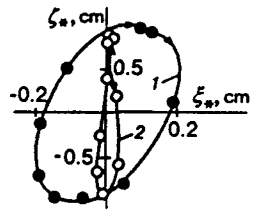

Fig. 4

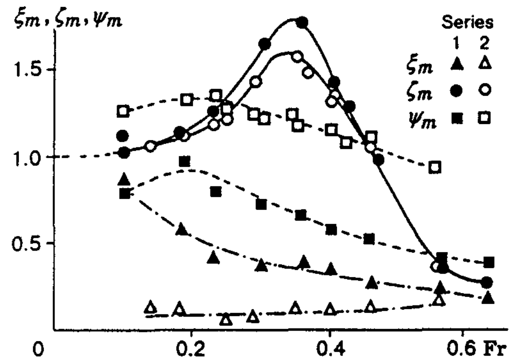

Fig. 5

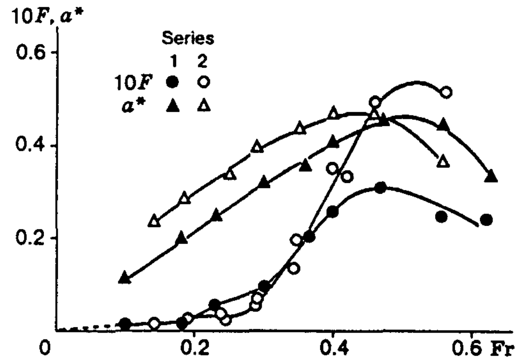

Fig. 6

that the wave amplitude changed simultaneously with the frequency. To study the effect of these parameters separately, we performed special tests in which the frequency was fixed, while the amplitude was varied. In particular, the drift velocity was found to be proportional to $(a/D)^{0.8}$ in the vicinity of the frequency given in the example, i.e., a considerable decrease in the drift was mainly due to a change in the wavelength.

In practice, the wavelength exceeds the cylinder diameter, the mean drift is insignificant, and it is important to decrease the amplitude of horizontal oscillations. In this respect, it is advantageous to reduce h. In Fig. 4, two trajectories of the body's center of mass in a moving coordinate system that moves in the direction of wave propagation at a constant drift velocity V are compared. Curve 1 was obtained in the first series of tests for $\Omega = 0.331$ rad/sec, $\lambda = 140$ cm, and $a = 0.41$ cm, while curve 2 was plotted for $\Omega = 0.321$ rad/sec, $\lambda = 143$ cm, and $a = 0.48$ cm in the second series of tests. In this example, the drift velocity is small: $V \approx 0.008$ cm/sec for both curves. It can be seen that a decrease in h by a factor of 5.7 reduces the amplitude of horizontal oscillations approximately by a factor of 5.

The character of trajectories in the moving coordinate system is of interest. For large h, they are similar to ellipses, whereas for small h, they look like Lissajous figures with one or more nodes. The character of trajectory 2 in Fig. 4 is indicative of the existence of the nonlinear effect of doubling of the horizontal oscillation frequency in comparison with the wave frequency. A Lissajous figure with two nodes was recorded in the second series of tests for $\Omega = 0.582$ rad/sec, $\lambda = 41.3$ cm, and $a = 0.94$ cm. The angular oscillation amplitude grows with decreasing h. However, the measuring equipment can be placed at the center of the cylinder.

Even for a very small e/l ratio ($e/l = 0.0005$), which occurred in the tests, the angular oscillations were stable and relatively small. The largest amplitude of the angular oscillations, equal to 0.239 rad, was recorded in the test whose conditions are given in the legend to curve 2 in Fig. 4.

More detailed information on the oscillation amplitudes for different degrees of freedom and also on the drift velocity is given in Figs. 5 and 6, where $\mathrm{Fr} = (2 + \varepsilon)D\Omega^2/\varepsilon g$, ξ_m and ζ_m are the horizontal and vertical oscillation amplitudes referred to the wave amplitude a, ψ_m is the angular oscillation amplitude referred to the wave slope $2\pi a/\lambda$, $F = V\Omega/\varepsilon g$, and $a_* = a/D$.

The vertical and angular oscillation amplitudes have maxima at the corresponding natural frequencies. The basic maximum of the drift velocity is at a higher frequency. Theoretically, the drift-velocity extrema are also possible at smaller frequencies (i.e., at longer wavelengths). This is associated with a complicated dependence of the cylinder resistance on the velocity of fluid flow past it. The resistance force has two components. One of them is due to fluid viscosity, while the other is due to the fact that the cylinder itself generates internal waves. Each component can change abruptly under certain conditions. In particular, it was noticed in the tests that the cylinder can generate divergent waves imposed on the fundamental wave. Such waves form under supercritical conditions for the propagation velocity of the second mode of intrinsic oscillations of a stratified fluid, namely, when the horizontal-motion velocity of the cylinder motion turns out to be higher than the propagation velocity of the second mode. The wave resistance of the cylinder changes sharply in transition through such a critical regime, which is one of the causes of the nonmonotone behavior of V.

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